Technical Notes

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Induction and Wave Drag on Long Cylindrical Satellites

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The induction drag and wave drag (due to Alfvén waves) on a long cylindrical satellite in the ionosphere moving at constant velocity V perpendicular to a constant magnetic field B_0 has been investigated by Chu and Gross.[1] Their work represents an extension of the work of Drell, Foley, and Ruderman[2] for a specific geometry and varying internal current. The notation used here is the same as that employed in Ref. 1.

In Ref. 1 an approximate calculation is made of the current drawn by the satellite from the plasma, and this current distribution is used to calculate an induced magnetic field using the Biot-Savart law. The drag is determined by numerical evaluation of integrals involving the induced fields which represent the power radiated by Alfvén waves. The conclusion of Ref. 1 is that if no dissipation of energy occurs in the wake, then the total power radiated is given by the product of the satellite velocity and the induced drag, and the Alfvén wave drag is included in the induced drag rather than an additional contribution.

The purpose of this Note is to make an analytical investigation of the integral which represents the radiated power as given in Ref. 1, and to show that certain conclusions reached in Ref. 1 are erroneous.

The power radiated from the satellite is given by Eq. (11) in Ref. 1 and is repeated below (correcting an inconsistency in axes)

$$P = V_A\mu \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[h_x^2 + 2h_y\left(\frac{B_0}{\mu}\right)\sin\alpha + h_y^2\right]dxdy \quad (1)$$

where

$$h_y = (2\pi)^{-1}\int_{-l/2}^{l/2} j(\xi)(x-\xi)[(x-\xi)^2 + y^2]^{-1}d\xi \quad (2)$$

and h_x is given by a similar expression with $(x - \xi)$ replaced by $-y$. The angle α is the angle between the direction of B_0(z-axis) and the direction of propagation of the Alfvén waves, which is quite small.

We consider first only the part of Eq. (1) which is linear in h_y. Substitution of Eq. (2) then provides the following integral P':

$$P' = B_0V\pi^{-1}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-l/2}^{l/2}\left[\frac{j(\xi)(x-\xi)d\xi}{(x-\xi)^2+y^2}\right]dxdy \quad (3)$$

The function $j(\xi)$ represents the current in the Alfvén wings and is assumed to be a piecewise continuous function which can be determined, although we will leave it arbitrary for the moment. If one attempts to integrate first with respect to x, it is easy to see that the integral does not exist. However, if the y integration is considered first and the integral rewritten

$$P' = B_0V\pi^{-1}\int_{-l/2}^{l/2}\int_{-\infty}^{\infty}\left\{\left[\int_{-\infty}^{\infty}\frac{dy}{y^2+(x-\xi)^2}\right]\times (x-\xi)dx\right\}j(\xi)d\xi \quad (4)$$

then it can be shown that the integral with respect to y does exist. Evaluation of this integral gives a value $\pi/|x - \xi|$, and hence Eq. (4) becomes

$$P' = B_0V\int_{-l/2}^{l/2}\left[\int_{-\infty}^{\infty}\frac{(x-\xi)}{|x-\xi|}dx\right]j(\xi)d\xi \quad (5)$$

The integral with respect to x still does not exist unless one defines it in terms of its principal value as follows:

$$P\int_{-\infty}^{\infty}\frac{(x-\xi)}{|x-\xi|}dx = \lim_{A\to\infty}\left[\int_{-A}^{\xi}(-dx) + \int_{\xi}^{A}dx\right] = -2\xi \quad (6)$$

Hence Eq. (5) can be written as

$$P' = -B_0V\int_{-l/2}^{l/2}2\xi j(\xi)d\xi \quad (7)$$

but $j(\xi) = (\frac{1}{2})(d/d\xi)[I(\xi)]$ as shown in Ref. 1 where $I(\xi)$ is the current in the satellite. Thus, substituting for $j(\xi)$ in Eq. (7) and then integrating by parts and noting that $I(\pm l/2) = 0$ gives

$$P' = VB_0\int_{-l/2}^{l/2}I(\xi)d\xi \equiv VD \quad (8)$$

where D is the induction drag as defined in Eq. (8). This result shows that the linear term in h_y that appears in Eq. (1) is the source of the induction drag for an arbitrary current distribution $I(\xi)$ in the satellite. The other two terms in Eq. (1) are of course both positive, and therefore, it is clear that the total drag is greater than the induction drag.

It was shown in Ref. 1 (see Fig. 6) that at low satellite altitudes the current distribution can be approximated very well by two straight line segments with the maximum value occurring at the point of zero potential. This can be expressed mathematically as

$$I = I_{\max}(l/2 \pm a)^{-1}(l/2 \pm x) \quad (9)$$

where the positive signs apply for $-l/2 \leq x \leq a$, and the negative signs for $a \leq x \leq l/2$. The induced magnetic field components can then be found by integration to be

$$h_x = I_{\max}(4\pi)^{-1}\left\{\left(\frac{l}{2}+a\right)^{-1}\tan^{-1}\frac{y(-a-l/2)}{y^2+(x-a)(x+l/2)} - \left(\frac{l}{2}-a\right)^{-1}\tan^{-1}\frac{y(a-l/2)}{y^2+(x-a)(x-l/2)}\right\} \quad (10)$$

$$h_y = I_{\max}(8\pi)^{-1}\left\{\left(\frac{l}{2}-a\right)^{-1}\log\frac{(x-l/2)^2+y^2}{(x-a)^2+y^2} - \left(\frac{l}{2}+a\right)^{-1}\log\frac{(x-a)^2+y^2}{(x+l/2)^2+y^2}\right\} \quad (11)$$

If Eq. (11) is substituted into the term linear in h_y in Eq. (1) then it can be shown that if integration with respect to

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y is done first followed by the principal part of the integral with respect to x, then $P' = (I_{max}B_0lV)/2$. This is the induction drag multiplied by the velocity as was shown to be generally true in Eq. (8). It should be noted that different results are obtained from those above if one integrates first with respect to x (taking the principal value) and then follows with integration over y. This is not unusual because in applying the principal value of an integral to a physical problem, it is done because the result does have a physical meaning.

The two terms in Eq. (1) involving h_x^2 and h_y^2 are generally several orders of magnitude smaller for typical satellite conditions than the induction drag. Since the work in Ref. 1 was done entirely from a numerical viewpoint (numerical evaluation of a divergent integral), and apparently the three terms were not considered individually, it is therefore apparent why Chu and Gross concluded that wave drag is part of the induction drag. However, the analytical investigation presented here makes it clear that their conclusion was not correct.

References

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[2] Drell, S. D., Foley, H. M., and Ruderman, M. A., "Drag and Propulsion of Large Satellites in the Ionosphere: An Alfvén Propulsion Engine in Space," *Journal of Geophysical Research*, Vol. 70, 1965, pp. 3131–3145.

Connection between Lift and Particle Displacement

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Introduction

IN the flow around a lifting body, the fluid particles which pass the low-pressure side go more quickly than those which pass the high-pressure side. If we assume the flow is uniform at infinity, the difference between the x-coordinates of two particles which start simultaneously far upstream, one passing far above the body, the other far below, results in a finite value at infinity downstream. This displacement has some correlation with the lift exerted on the body. Recently, Corrsin has made a conjecture on the connection between the lift and the particle displacement.[1] His analytical result is in good agreement with his numerical calculation. His analysis, however, has been based on an assumption that the changes of y-coordinates of the fluid particles can be neglected. This is not true in large scale of time. Therefore, there has been left some ambiguity in the validity of his result far downstream.

In the present paper, it is shown that his conjecture can be proved without the assumption stated previously, by use of the stream function as an integral of the equations of motion of the fluid particle. Similar formulation can be found in the analyses of drift in the flow with no circulation around a circular cylinder by Darwin and around a sphere by Lighthill.[2,3] In the following analysis, the prime is used in order to denote the dimensional quantities which shall be nondimensionalized later.

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Formulation of the Problem

We consider a cylindrical body with the circulation Γ in a uniform flow with the velocity U. If we restrict our consideration to the region sufficiently far from the body, we need consider only the lifting vortex filament located at the origin, because the effect of the shape of the body decreases with the distance from the body r' as r'^{-2}.

The flowfield is described in terms of the stream function ψ' as

$$\psi' = Ur' \sin\theta + (\Gamma/2\pi) \ln r' \tag{1}$$

$$dr'/dt' = (1/r')(\partial\psi'/\partial\theta) = U \cos\theta \tag{2}$$

$$\frac{d\theta}{dt'} = -\frac{1}{r'}\frac{\partial\psi'}{\partial r'} = -\frac{1}{r'}\left(U \sin\theta + \frac{\Gamma}{2\pi}\frac{1}{r'}\right) \tag{3}$$

where (r',θ) is the polar coordinate with the origin at the vortex, t' the time, dr'/dt' and $d\theta/dt'$ the velocity components of the fluid particle in the Lagrangian sense. It is convenient to use the dimensionless variables r, t, and ψ, defined by

$$r = (2\pi U/\Gamma)r' \tag{4a}$$

$$t = (2\pi U^2/\Gamma)t' \tag{4b}$$

$$\psi = (2\pi/\Gamma)\psi' \tag{4c}$$

Then, Eqs. (1–3) become

$$\psi = r \sin\theta + \ln r \tag{5}$$

$$dr/dt = \cos\theta \tag{6}$$

$$d\theta/dt = -(1/r)(\sin\theta + 1/r) \tag{7}$$

The Eqs. (6) and (7) constitute a set of equations which describe the motion of the fluid particle, and the stream function is one of its integrals

$$\psi = r \sin\theta + \ln r = \alpha \tag{8}$$

where the parameter α is the constant of motion and its value can be regarded as a label for each streamline.

Eliminating θ from Eqs. (7) and (8), we obtain

$$dr/dt = \pm[1 - (1/r^2)(\alpha - \ln r)^2]^{1/2} \tag{9}$$

Integrating this equation, we get the solution

$$t = \pm \int \frac{r dr}{[r^2 - (\alpha - \ln r)^2]^{1/2}} \tag{10}$$

This is the drift function defined by Lighthill.[3]

Here we define a Cartesian coordinate (x,y) by

$$x = r \cos\theta \quad \text{and} \quad y = r \sin\theta \tag{11}$$

The quantity we are interested in is the displacement of the particle in the x direction, referred to axes in which the infinite parts of the fluids are at rest. For this, what is required is $X = x - t$. This quantity has been named the drift by Darwin.[2] Combining Eqs. (6) and (9) with (11), we get

$$x = [r^2 - (\alpha - \ln r)^2]^{1/2} \tag{12}$$

From Eqs. (10) and (11) we have

$$X = [r^2 - (\alpha - \ln r)^2]^{1/2} - \int_{r_1}^{r} \frac{r dr}{[r^2 - (\alpha - \ln r)^2]^{1/2}} = \int_{r_1}^{r} \frac{(\alpha - \ln r) dr}{r[r^2 - (\alpha - \ln r)^2]^{1/2}} \tag{13}$$

where we are considering the fluid particles which start from